Study of Laguerre Gaussian Beam Induced Azimuthal Doppler Shift using Saturation Absorption Spectroscopy

J. Anupriya, Nibedita Ram, M. Pattabiraman

Department of Physics, Indian Institute of Technology Madras,
Chennai 600 036, India

Laguerre Gaussian beam

- Doughnut shape intensity distribution with zero intensity at the centre
- Has a helical wave front
- carries an OAM (Orbital Angular Momentum)
- Has a non vanishing Azimuthal phase dependence

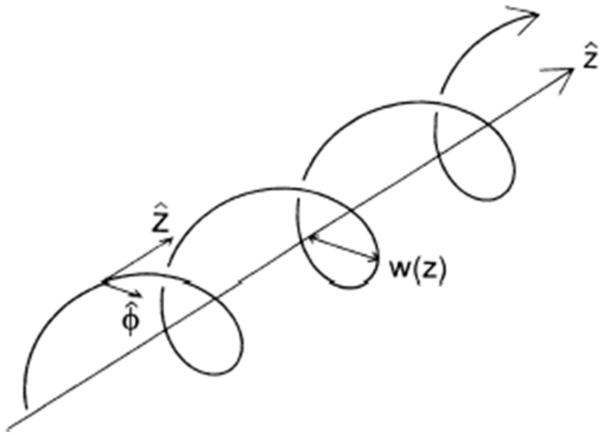


Figure 1: propagation of LG beam (Courtesy: [1])

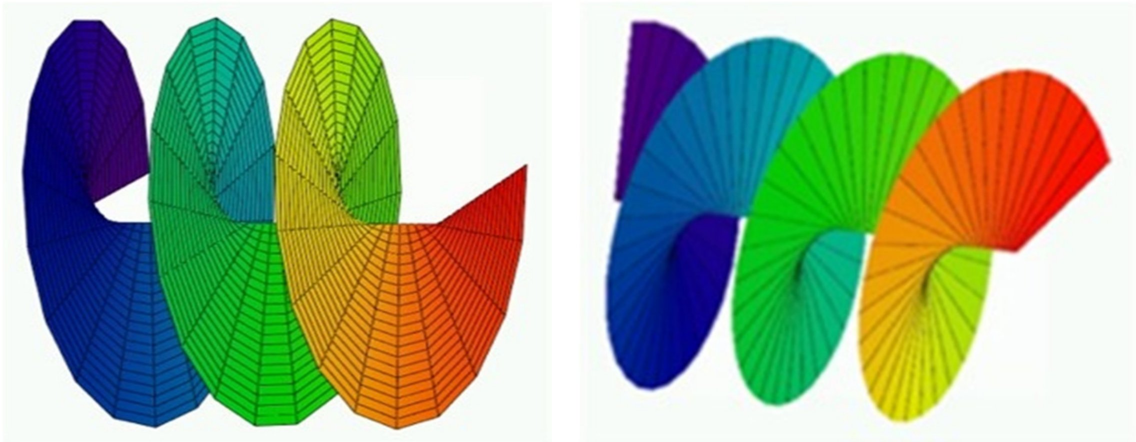


Figure 2: LG_0^{+1} and LG_0^{-1} (courtesy: [2])

[1]"Orbital Angular momentum of light and the transformation of the Laguerre Gaussian laser modes' L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw and J. P. Woerdman, Phys. Rev. A **45**,11 (1992).

[2]"departments.colgate.edu/physics/research/optics/oamgp/gp.htm"

Laguerre Gaussian mode

The electric field vector associated with a LG mode:

$$E(\mathbf{R}) = i\left[a\hat{e}\varepsilon_{klp}(\mathbf{R})e^{i\Theta_{klp}(\mathbf{R})} - H.c\right]$$

$\varepsilon_{klp}(\mathbf{R}) \rightarrow$ **mode amplitude and**

$\Theta_{klp}(\mathbf{R}) \rightarrow$ **phase factor of the electric field respectively**

The field amplitude satisfying the wave equation in the paraxial approximation is given by:

$$u_{LG_p}^{l}(r,\theta,\phi) \propto \text{Laguerre polynomial} \times \text{Gaussian envelope} \times \exp[il\phi]$$

$$\propto \varepsilon_{klp}(\mathbf{R})e^{i\Theta_{klp}(\mathbf{R})} - H.c$$

The Doppler shift associated with the LG beam:

$$\delta = \nabla\Theta.\mathbf{v} = \delta_{axial} + \delta_{radial} + \delta_{azimuthal}$$

$$\delta_{axial} = \left\{k + \frac{kr^2}{2(z+z_R^2)}\left[1 - \frac{2z^2}{z^2+z_R^2}\right] + \frac{(2p+l+1)}{z^2+z_R^2}z_R\right\}v_z$$

$$\delta_{radial} = \left[\frac{krz}{z^2+z_R^2}\right]v_r \text{ and } \delta_{azimuthal} = \frac{lv_\phi}{r}$$

$\hat{e} \rightarrow$ **is the polarization vector in the X-Y plane,**

$l \rightarrow$ **Azimuthal mode index**

$p \rightarrow$ **Radial mode index**

$k \rightarrow$ **wave vector**

$\delta_{azimuthal}$ **is responsible for the additional shift in the atomic resonance**

"Atomic motion in light beams possessing orbital angular momentum", W. L. Power, L. Allen, M. Babiker and V. E. Lembessis, Phys. Rev. A **52**,1 (1995).

Azimuthal Doppler shift

The normalized Lorentzian line shape factor describing the lineshape of the atomic transition is given by:

$$L(x) = \frac{1}{2\pi}\frac{\gamma}{x^2 + \left(\frac{\gamma}{2}\right)^2}$$

$A \longrightarrow$ is a constant
$\gamma \longrightarrow$ Full width at half maximum
$x \longrightarrow$ is the detuning

The homogeneous line shape is given by:

$$h(\delta' - \delta, r) = AI_1(r)I_2(r)L(\delta' - \delta)$$

$$I_j(r) = I_{01} r^{2|l_j|} e^{-\frac{2r^2}{W^2(z)}}$$

The total signal of the atomic vapour is given by:

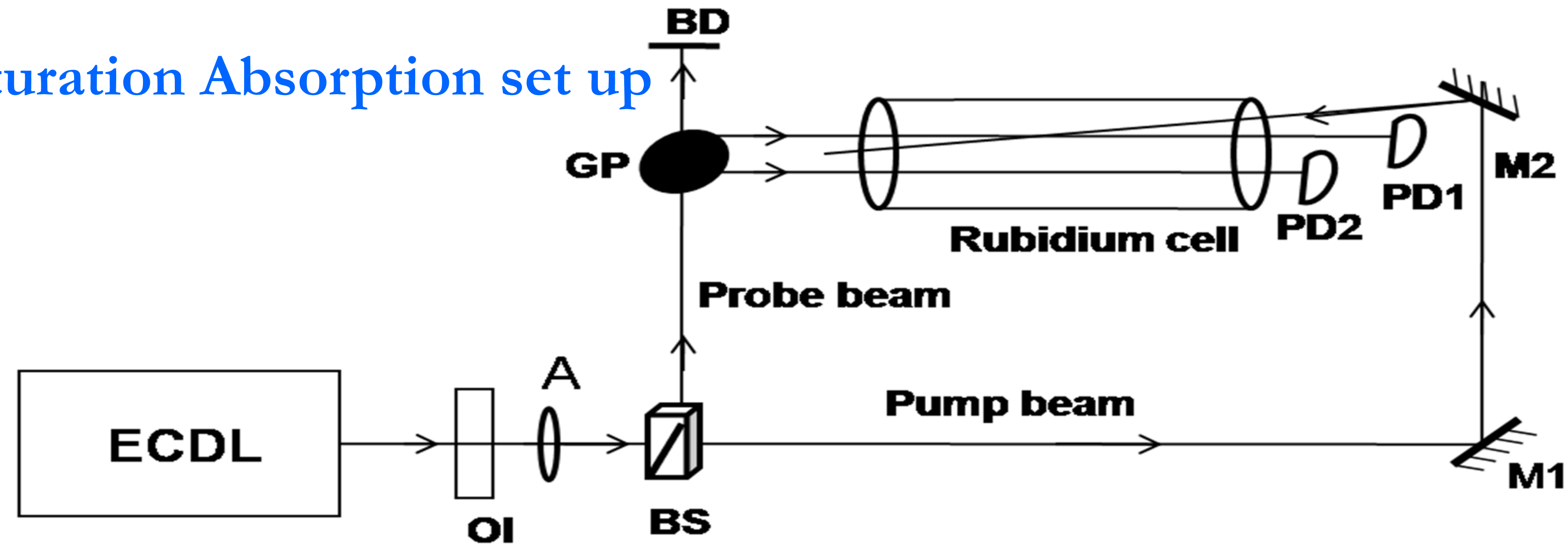
$$S(\delta) = \int_0^{\infty} 2\pi r dr \int_{-\infty}^{+\infty} h(\delta' - \delta, r) W(V_\phi) dV_\phi$$

$W(z) \rightarrow$ Propagation distance dependent beam radius
$W(V_\phi) \rightarrow$ is the velocity distribution at thermal equilibrium at temperature T
$I_j \rightarrow$ The intensity distribution of field j
$r \rightarrow$ is the atom radial position

$$S(\delta) = C\int_{-\infty}^{+\infty} L(\delta' - \delta)\left[\frac{\alpha\delta^2}{(l_1 - l_2)^2} + \frac{4}{W^2(z)}\right]^{-(|l_1| + |l_2| + 3/2)} d\delta'$$

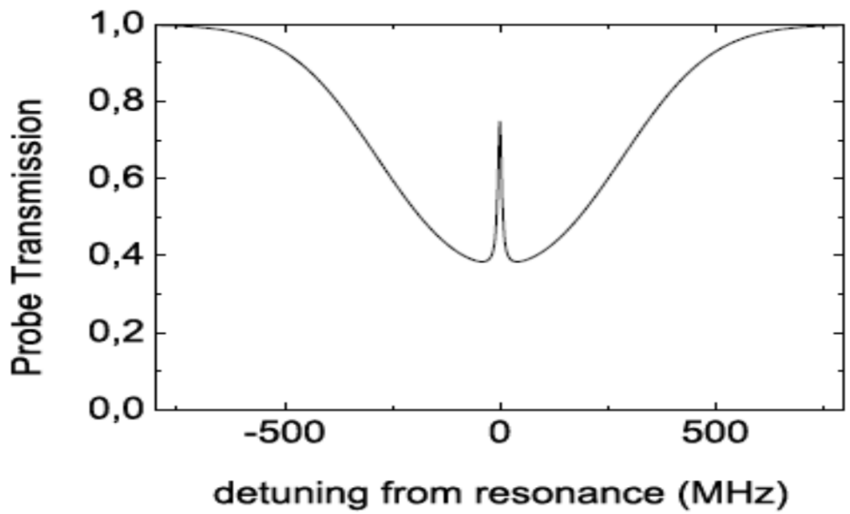
"Spectroscopic observation of the rotational Doppler effect", S. Barreiro, J.W.R. Tabosa, H. Failache, and A. Lezama Phys. Rev. Lett. **97**,113601 (2006).

Saturation Absorption set up



ECDL: External Cavity Diode Laser, OI: Optical Isolator, GP: Glass Plate, A: Aperture, M: Mirror, BS: 50-50 Beam Splitter Cube, BD: Beam Dump, PD: Photo Detector.

Lamb Dip



The Doppler shift (Axial and Radial) associated with the Gaussian beam (with the leading term of kV_z) cancels out in the saturation absorption set up with a counter-propagating pump and probe beam.

Doppler shift associated with LG beam

$$\delta_{LG_1} = \left\{ k + \frac{kr^2}{2\left(z+z_R^2\right)}\left[1 - \frac{2z^2}{z^2+z_R^2}\right] + \frac{(2p+l+1)}{z^2+z_R^2} z_R \right\} v_z + \left[\frac{krz}{z^2+z_R^2}\right] v_r + \frac{l_1 v_\phi}{r}$$

$$\delta_{LG_2} = -\left\{ k + \frac{kr^2}{2\left(z+z_R^2\right)}\left[1 - \frac{2z^2}{z^2+z_R^2}\right] + \frac{(2p+l+1)}{z^2+z_R^2} z_R \right\} v_z - \left[\frac{krz}{z^2+z_R^2}\right] v_r - \frac{l_2 v_\phi}{r}$$

δ_{LG_1} and δ_{LG_2} are the Doppler shift associated with the probe and the pump beam with Azimuthal charge index of l_1 and l_2 respectively.

$$\delta_{Total} = \delta_{LG_1} + \delta_{LG_2} = \frac{l_1 - l_2}{r} V_\phi$$

- The rotational Doppler shift has been observed using interferometric technique [1] and recently the Azimuthal Doppler shift was measured using the Hanle-EIT configuration with a co-propagating probe and pump beam [2].
- We have also made a spectroscopic observation of the Azimuthal Doppler shift using a saturation absorption set up with a counter-propagating pump and probe beam.

[1] "Manifestation of the rotational Doppler effect by use of an off-axis optical vortex beam", I. V. Basistiy, V. V. Slyusar, M. S. Soskin, M. V. Vasnetsov and A. Ya. Bekshaev, Opt. Lett. **28**, 1185 (2003)

[2]"Spectroscopic observation of the rotational Doppler effect", S. Barreiro, J.W.R. Tabosa, H. Failache, and A. Lezama Phys. Rev. Lett. **97**,113601 (2006).

Generation of LG beam:

- The simplest form of LG beam can be represented as:

$$E(r,\phi,z) = E_0 e^{-ik_z z} e^{il\phi}$$

- Plane wave U propagating obliquely to the Z axis:

$$e^{ik_x x - ik_z z}$$

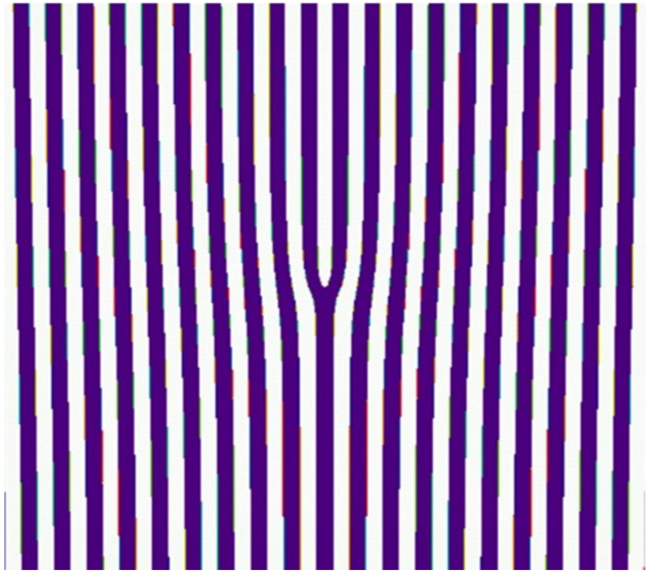
- I=The sum of the amplitude function.

If the Recording device is assumed to be located at Z=0, then we have:

$$I = I_1 + I_2 + 2\,\mathrm{Re}(\text{Correlation Function}) = 1 + |E_0|^2 + 2E_0 \cos(k_x x - l\phi)$$

- The Fourier Transform of I gives the Transmittance function which can be used to create the Diffraction Grating which is called as the Computer Generated Hologram (CGH)

LG beams can be generated by using a CGH:



CGH

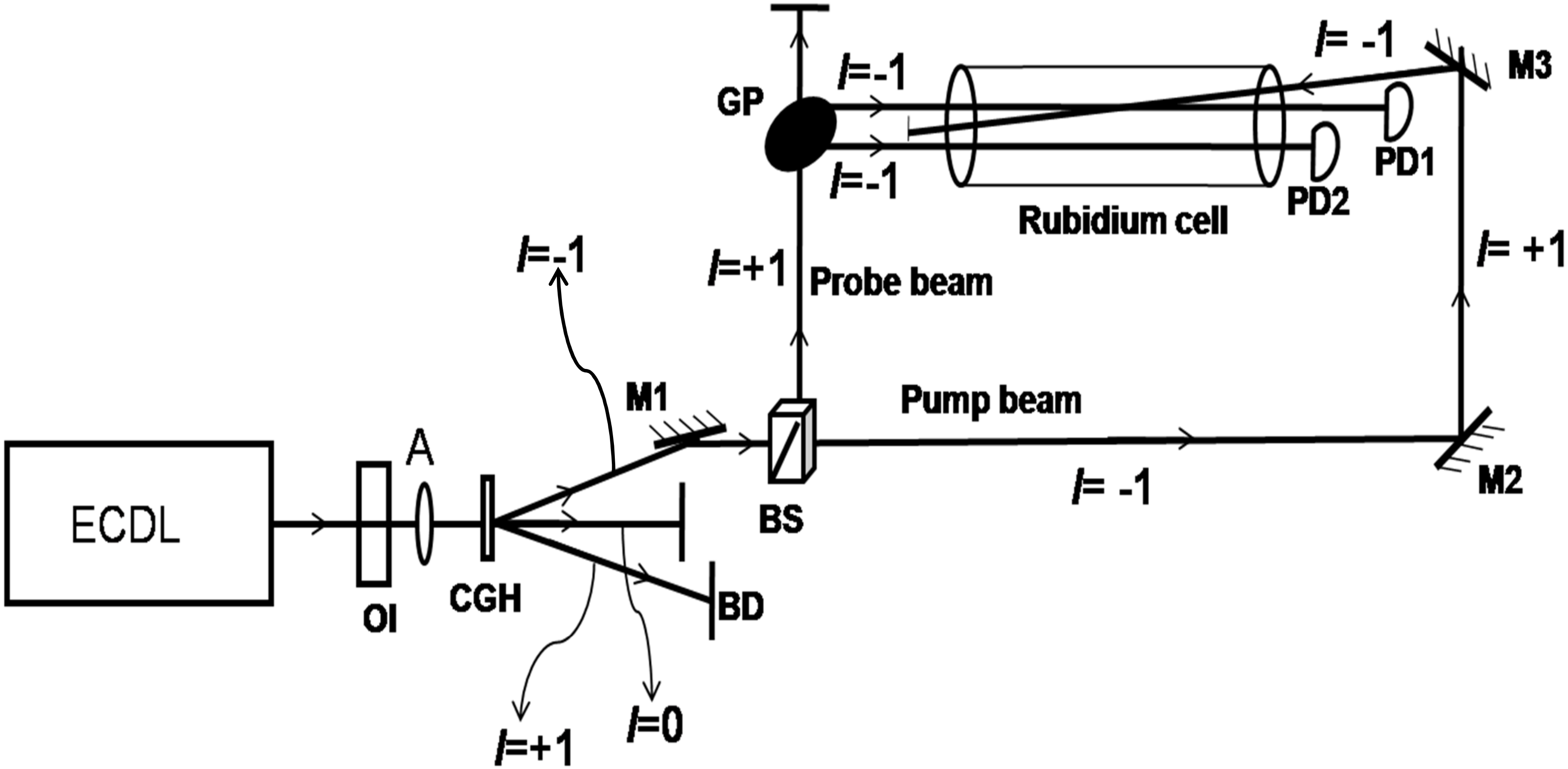
$$LG_{p=0}^{l=+1} \quad LG_{p=0}^{l=0} \quad LG_{p=0}^{l=-1}$$

Picture of the generated LG beam

These holograms are produced by calculating the interference pattern of a Gaussian and a LG beam with *l*=+1

The computer generated holograms were obtained from: "departments.colgate.edu/physics/research/optics/oamgp/homemadedos.htm"

Counter-propagating Set-up



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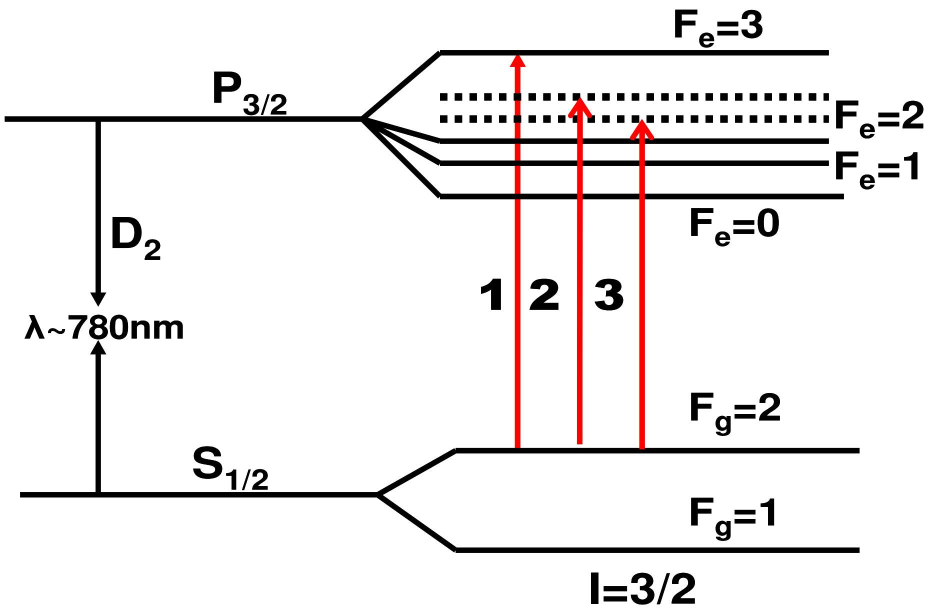
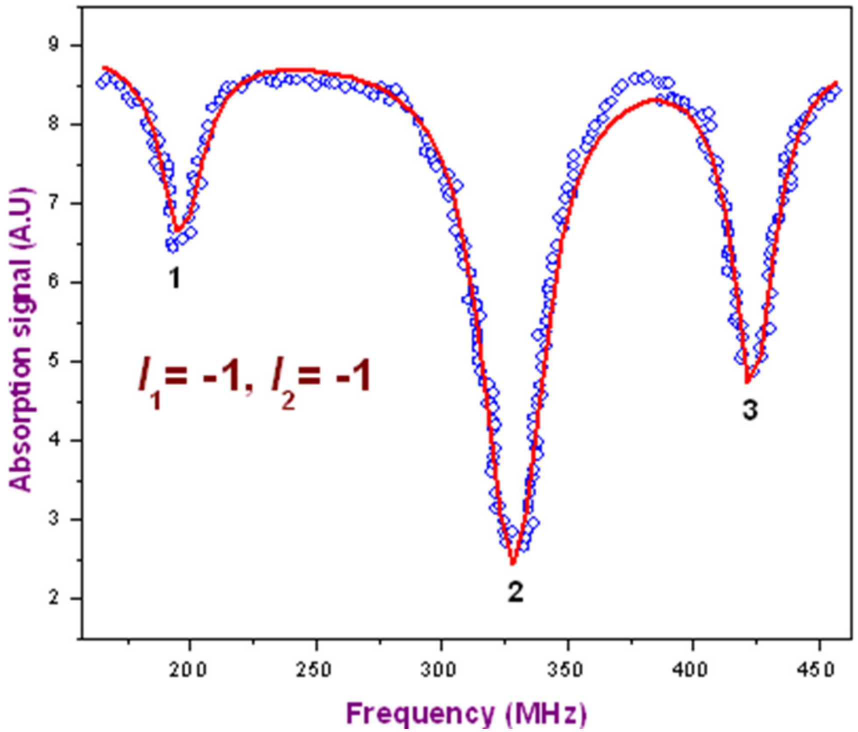
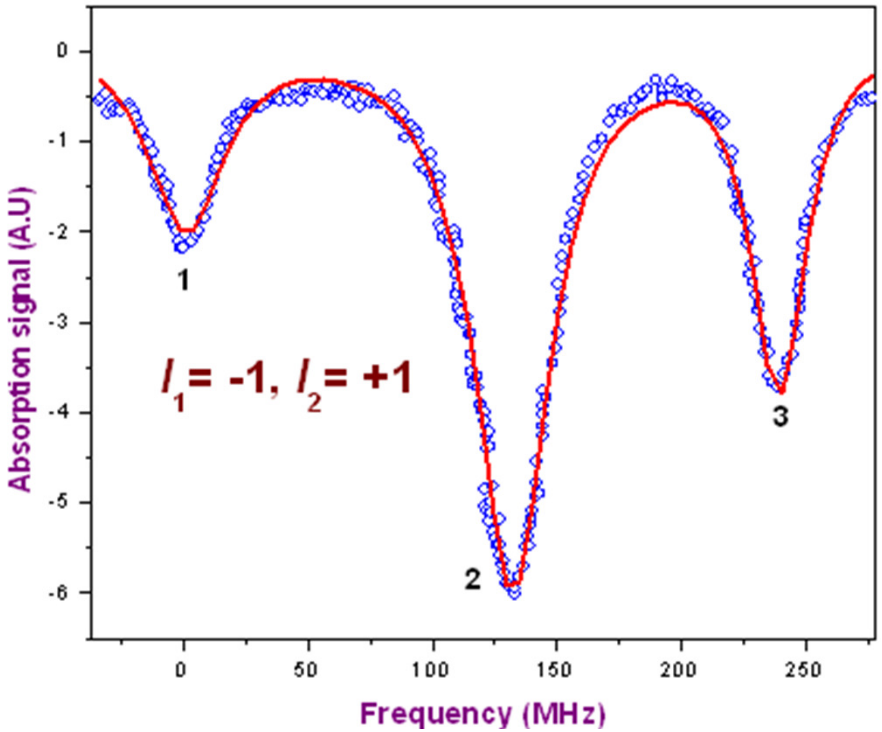
The Energy level diagram of Rb^{87}



The Saturation Absorption spectrum





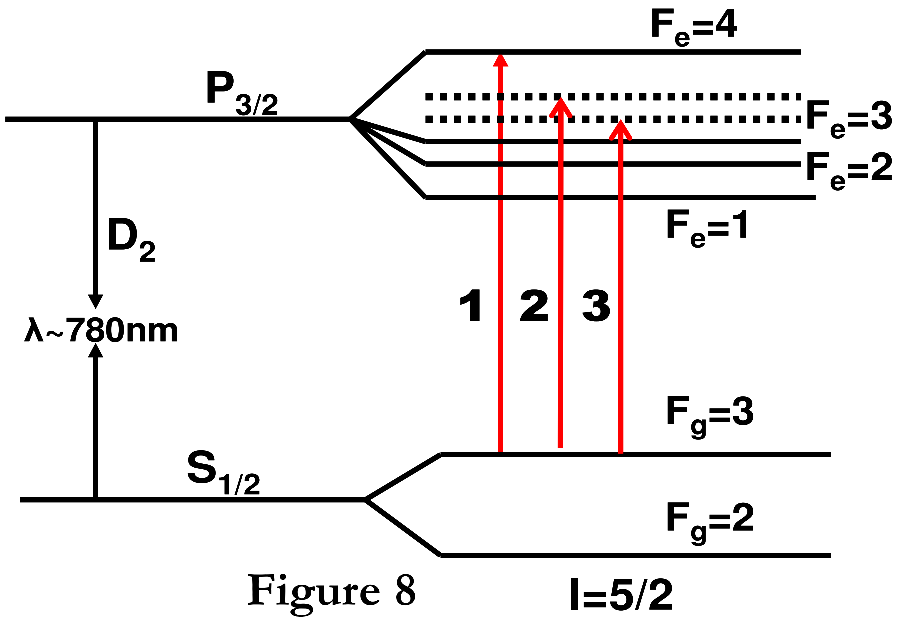
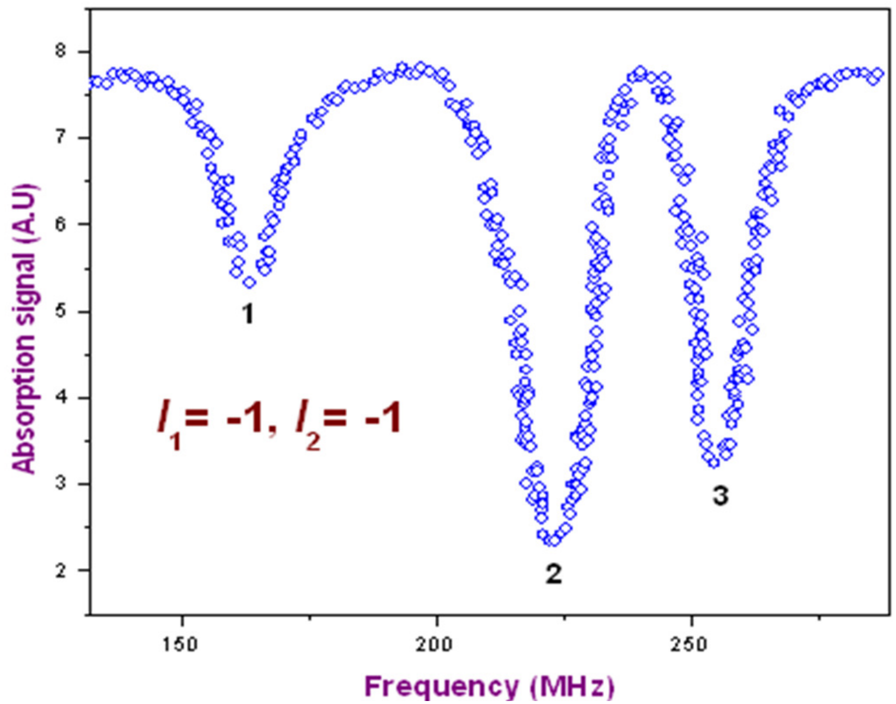


Figure 8

The Energy level diagram of Rb^{85}

The Saturation Absorption spectrum



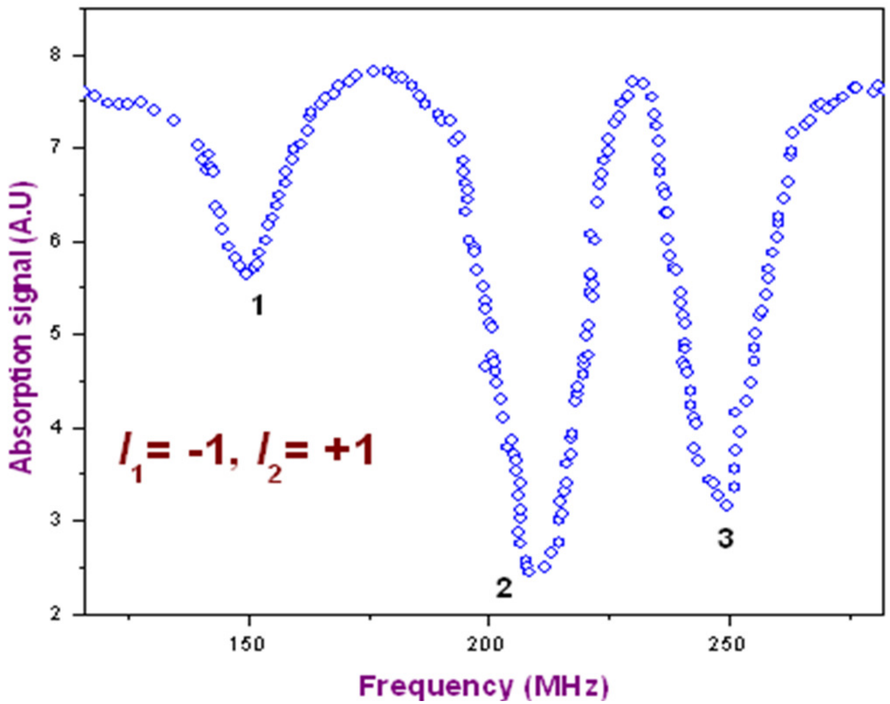


Table of Line widths (MHz)

Rb^{87}	$l_1 = -1, l_2 = -1$	$l_1 = -1, l_2 = +1$	Azimuthal Doppler shift
$F_g=2 \rightarrow F_e=(2,3)$	29.9693	36.9102	6.9409
$F_g=2 \rightarrow F_e=(1,3)$	20.325	25.991	5.666

Rb^{85}	$l_1 = -1, l_2 = -1$	$l_1 = -1, l_2 = +1$	Azimuthal Doppler shift
$F_g=3 \rightarrow F_e=4$	12.1625	16.452	4.2895
$F_g=3 \rightarrow F_e=(3,4)$	18.8613	23.3096	4.4483
$F_g=3 \rightarrow F_e=(2,4)$	14.2339	23.0600	8.8261

The Azimuthal Doppler shift has been determined using the saturation absorption spectroscopy in the counter-propagating mode.