Study of Laguerre Gaussian Beam Induced Azimuthal Doppler Shift using Saturation Absorption Spectroscopy

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Laguerre Gaussian beam

• Doughnut shape intensity distribution with zero intensity at the centre
• Has a helical wave front
• Carries an OAM (Orbital Angular Momentum)
• Has a non-vanishing Azimuthal phase dependence


[2]“departments.colgate.edu/physics/research/optics/oamgp/gp.htm”
Laguerre Gaussian mode

The electric field vector associated with a LG mode:

\[ E(R) = i \left[ a^\ast \epsilon_{klp}(R) e^{i\Theta_{klp}(R)} - H.c \right] \]

The field amplitude satisfying the wave equation in the paraxial approximation is given by:

\[ u_{LGp}^l(r, \theta, \phi) \propto \text{Laguerre polynomial} \times \text{Gaussian envelope} \times \exp[il\phi] \]

\[ \propto \epsilon_{klp}(R) e^{i\Theta_{klp}(R)} - H.c \]

The Doppler shift associated with the LG beam:

\[ \delta = \nabla \Theta \cdot \mathbf{v} = \delta_{\text{axial}} + \delta_{\text{radial}} + \delta_{\text{azimuthal}} \]

\[ \delta_{\text{axial}} = \left\{ k+ \frac{kr}{2(z+z_R^2)} \left[ 1 - \frac{2z^2}{z^2 + z_R^2} \right] + \frac{(2p+l+1)}{z^2 + z_R^2} z_R \right\} v_z \]

\[ \delta_{\text{radial}} = \frac{krz}{z^2 + z_R^2} v_r \text{ and } \delta_{\text{azimuthal}} = \frac{lv_\phi}{r} \]

\[ \hat{e} \rightarrow \text{is the polarization vector in the X-Y plane,} \]
\[ l \rightarrow \text{Azimuthal mode index} \]
\[ p \rightarrow \text{Radial mode index} \]
\[ k \rightarrow \text{wave vector} \]

Azimuthal Doppler shift

The normalized Lorentzian line shape factor describing the lineshape of the atomic transition is given by:

\[ L(x) = \frac{1}{2\pi} \frac{\gamma}{x^2 + (\frac{\gamma}{2})^2} \]

\( A \rightarrow \) is a constant
\( \gamma \rightarrow \) Full width at half maximum
\( x \rightarrow \) is the detuning

The homogeneous line shape is given by:

\[ h(\delta' - \delta, r) = AI_1(r)I_2(r)L(\delta' - \delta) \]

The total signal of the atomic vapour is given by:

\[ S(\delta) = \int_{0}^{\infty} \int_{-\infty}^{\infty} h(\delta' - \delta, r)W(V_\phi)dV_\phi \]

\[ I_j(r) = I_{01}r^{2|l_j|}e^{-\frac{2r^2}{W^2(z)}} \]

\( W(z) \rightarrow \) Propagation distance dependent beam radius
\( W(V_\phi) \rightarrow \) is the velocity distribution at thermal equilibrium at temperature T
\( I_j \rightarrow \) The intensity distribution of field \( j \)
\( r \rightarrow \) is the atom radial position

\[ S(\delta) = C \int_{-\infty}^{+\infty} L(\delta' - \delta) \left[ \frac{\alpha \delta^2}{(l_1 - l_2)^2} + \frac{4}{W^2(z)} \right]^{-\frac{1}{2}}d\delta' \]

The Doppler shift (Axial and Radial) associated with the Gaussian beam (with the leading term of $kV_z$) cancels out in the saturation absorption set up with a counter-propagating pump and probe beam.
The rotational Doppler shift has been observed using interferometric technique [1] and recently the Azimuthal Doppler shift was measured using the Hanle-EIT configuration with a co-propagating probe and pump beam [2].

We have also made a spectroscopic observation of the Azimuthal Doppler shift using a saturation absorption set up with a counter-propagating pump and probe beam.

\[ \delta_{LG_1} = \left\{ k^+ \frac{kr^2}{2(z+z_R^2)} \left[ 1 - \frac{2z^2}{z^2 + z_R^2} \right] + \frac{(2p+l+1)}{z^2 + z_R^2} z_R \right\} v_z + \left[ \frac{krz}{z^2 + z_R^2} \right] v_r + \frac{l_1 v_\phi}{r} \]

\[ \delta_{LG_2} = \left\{ k^+ \frac{kr^2}{2(z+z_R^2)} \left[ 1 - \frac{2z^2}{z^2 + z_R^2} \right] + \frac{(2p+l+1)}{z^2 + z_R^2} z_R \right\} v_z - \left[ \frac{krz}{z^2 + z_R^2} \right] v_r - \frac{l_2 v_\phi}{r} \]

\[ \delta_{Total} = \delta_{LG_1} + \delta_{LG_2} = \frac{l_1 - l_2}{r} V_\phi \]

\[ \delta_{LG_1} \text{ and } \delta_{LG_2} \text{ are the Doppler shift associated with the probe and the pump beam with Azimuthal charge index of } l_1 \text{ and } l_2 \text{ respectively.} \]

• The rotational Doppler shift has been observed using interferometric technique [1] and recently the Azimuthal Doppler shift was measured using the Hanle-EIT configuration with a co-propagating probe and pump beam [2].

• We have also made a spectroscopic observation of the Azimuthal Doppler shift using a saturation absorption set up with a counter-propagating pump and probe beam.


Generation of LG beam:

- The simplest form of LG beam can be represented as:

\[ E(r, \phi, z) = E_0 e^{-ik_z z} e^{il\phi} \]

- Plane wave \( U \) propagating obliquely to the Z axis:

\[ e^{ik_x x - ik_z z} \]

- \( I \)=The sum of the amplitude function.

If the Recording device is assumed to be located at \( Z=0 \), then we have:

\[ I = I_1 + I_2 + 2 \text{Re(Correlation Function)} = 1 + |E_0|^2 + 2E_0 \cos(k_x x - l\phi) \]

- The Fourier Transform of \( I \) gives the Transmittance function which can be used to create the Diffraction Grating which is called as the Computer Generated Hologram (CGH)
LG beams can be generated by using a CGH:

These holograms are produced by calculating the interference pattern of a Gaussian and a LG beam with $l=+1$

The computer generated holograms were obtained from:
“departments.colgate.edu/physics/research/optics/oamgp/homemadedos.htm”
Counter-propagating Set-up

The Energy level diagram of Rb^{87}

The Saturation Absorption spectrum

I = 3/2

$\lambda \sim 780\text{nm}$

$F_e = 3$

$F_e = 2$

$F_e = 1$

$F_g = 2$

$F_g = 1$

$P_{3/2}$

$S_{1/2}$

$D_2$

$l_1 = -1, l_2 = -1$

$l_1 = -1, l_2 = +1$
The Energy level diagram of $^{85}\text{Rb}$

$D_2$  
$\lambda \approx 780\text{nm}$  

$P_{3/2}$  
$F_e=4$  
$F_e=3$  
$F_e=2$  
$F_e=1$  
$S_{1/2}$  
$F_g=2$  
$I=5/2$

Figure 8

The Saturation Absorption spectrum

$|l_1|=1$, $|l_2|=1$

$|l_1|=1$, $|l_2|=+1$
The Azimuthal Doppler shift has been determined using the saturation absorption spectroscopy in the counter-propagating mode.

<table>
<thead>
<tr>
<th></th>
<th>$I_1 = -1, I_2 = -1$</th>
<th>$I_1 = -1, I_2 = +1$</th>
<th>Azimuthal Doppler shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_g=2 \rightarrow F_e=(2,3)$</td>
<td>29.9693</td>
<td>36.9102</td>
<td>6.9409</td>
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<tr>
<td>$F_g=2 \rightarrow F_e=(1,3)$</td>
<td>20.325</td>
<td>25.991</td>
<td>5.666</td>
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<td>$F_g=3 \rightarrow F_e=4$</td>
<td>12.1625</td>
<td>16.452</td>
<td>4.2895</td>
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<td>$F_g=3 \rightarrow F_e=(3,4)$</td>
<td>18.8613</td>
<td>23.3096</td>
<td>4.4483</td>
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<tr>
<td>$F_g=3 \rightarrow F_e=(2,4)$</td>
<td>14.2339</td>
<td>23.0600</td>
<td>8.8261</td>
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</tbody>
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